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Using a squeezed field to protect two-atom entanglement against spontaneous emissions

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Abstract

Tunable interaction between two atoms in a cavity is realized by interacting the two atoms with an extra controllable single-mode squeezed field. Such a controllable interaction can be further used to control entanglement between the two atoms against amplitude damping decoherence caused by spontaneous emissions. For the independent amplitude damping decoherence channel, entanglement will be lost completely without controls, while it can be partially preserved by the proposed strategy. For the collective amplitude damping decoherence channel, our strategy can enhance the entanglement compared with the uncontrolled case when the entanglement of the uncontrolled stationary state is not too large.

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1. Introduction

Quantum entanglement [1–6] is a fundamental property of multi-body quantum systems that shows the non-local feature of quantum states. Quantum entanglement has been commonly recognized to be an essential physical resource in the implementation of high-speed quantum computation and high-security quantum communication.

Many efforts have been made to create entanglement between decoupled quantum systems. One natural way is to introduce a simple intermediate device [7–11], e.g., a single-mode field or an additional particle, whose coherent interactions with the systems lead to their indirect interactions with each other. The intermediate device can also be measured to extract information about the quantum systems for quantum feedback controls [12–14] to manipulate the entanglement dynamics. One may also utilize a dissipative environment [15–18], e.g., a collective decoherence environment, to generate entanglement, interacted with which the system irreversibly decays to a stationary entangled state.

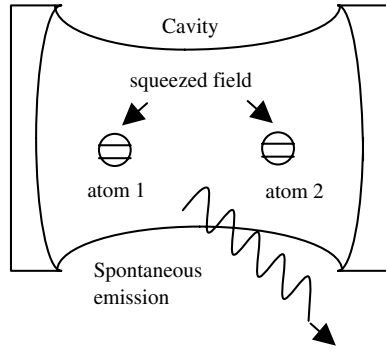


Figure 1. Two atoms undergoing decoherence caused by spontaneous emissions interact with a single-mode squeezed field in a cavity.

However, in most circumstances, quantum entanglement tends to be destructed in environments [19–22]. For example, independent decoherence channels always lead to disentanglement [23] that is not recoverable by local operations and classical communications.

Generally, non-local operations are required to effectively protect entanglement. However, a non-local Hamiltonian generated from the internal interaction between quantum systems, e.g., the dipole–dipole interaction between two atoms via the vacuum, is sometimes not a good choice, because disentanglement can also be induced by decoherence under these interactions.

This paper introduces a single-mode squeezed field in a quantum cavity to realize non-local controllable interactions between two identical atoms in the weak coupling regime. By altering the parameter amplification coefficient of the squeezed field, one can continuously adjust the coupling strengths between atoms, which can be further used to control the final entanglement between the two atoms in the presence of decoherence. It should be pointed out that there is another interesting work on coupling the two atoms via the squeezed vacuum [24]. Compared with the squeezed vacuum, the auxiliary squeezed field in the cavity is more controllable, which would be helpful to control the stationary concurrence.

The paper is organized as follows: the physical model applied in the paper is formulated in section 2. Entanglement control strategies are discussed for two-atom independent amplitude damping decoherence channels, collective amplitude damping decoherence channels, and their mixture, respectively in sections 3–5. Conclusions and a forecast of the future work are drawn in section 6.

2. Model formulation

Consider the system of two identical two-level atoms interacting with a squeezed single-mode field in a quantum cavity (see figure 1). The total Hamiltonian of the atoms and the cavity mode can be described as below with \hbar assumed to be 1 without loss of generality:

$$H_{AC} = \omega_c a^\dagger a + \frac{\omega_a}{2} \sum_{i=1}^2 \sigma_z^{(i)} + \sum_{i=1}^2 (\epsilon^{(i)} a \sigma_+^{(i)} + \epsilon^{(i)*} a^\dagger \sigma_-^{(i)}) + (\xi e^{-i\Omega t} a^{\dagger 2} + \xi^* e^{i\Omega t} a^2), \quad (1)$$

where the first two terms describe the free Hamiltonians of the cavity mode and the atoms, ω_c is the frequency of the cavity mode, and ω_a is the inherent frequency of the atom corresponding to the energy separation between the ground state and the excited state of each atom, a is the annihilation operator of the cavity mode, and $\sigma_z^{(i)}$, $i = 1, 2$ is the z -axis Pauli operator of the

i th atom. The third term represents the interaction between the atoms and the cavity mode, in which $\sigma_{\pm}^{(i)} = \sigma_x^{(i)} \pm i\sigma_y^{(i)}$, $i = 1, 2$, are the ladder operators of the i th atom. The complex coefficient,

$$\epsilon^{(i)} = \vec{\mu} \cdot \vec{g}(\vec{r}^{(i)}),$$

is the inner product of the transition dipole moment $\vec{\mu}$ of each atom and the coupling constant

$$\vec{g}(\vec{r}^{(i)}) = \left(\frac{\omega_c}{2\epsilon_0 V} \right)^{\frac{1}{2}} \hat{e}_{\vec{k}} e^{i\vec{k} \cdot \vec{r}^{(i)}}, \quad (2)$$

where $\vec{r}^{(i)}$ is the position of the i th atom, \vec{k} and $\hat{e}_{\vec{k}}$ are the wave vector and unit polarization vector of the cavity mode, and V is the normalization volume of the cavity mode. The last term is the Hamiltonian of the squeezed cavity mode, where the parameter amplification coefficient ξ and the frequency Ω are continuously tunable. Such a manipulable standing squeezed field in a high- Q cavity is realizable by squeezed state engineering developed recently [26, 27]. Roughly speaking, a three-level atom in a ladder configuration is introduced to interact with the cavity mode. In addition, a classical field is used to manipulate the three-level atom, through which one can continuously adjust the squeezed coefficient ξ and the frequency Ω .

In the weak coupling regime, i.e., $\Delta = \omega_a - \omega_c$ and $|\xi| \gg |\epsilon^{(i)}|$, H_{AC} can be diagonalized by the following unitary transform [28]:

$$U = \exp \left[\frac{1}{\Delta} \sum_{i=1}^2 (\epsilon^{(i)} a \sigma_+^{(i)} - \epsilon^{(i)*} a^\dagger \sigma_-^{(i)}) \right],$$

which, by taking the first-order approximation of $\epsilon^{(i)}/\Delta$, gives the following expression:

$$\begin{aligned} H_{AC} &\approx U H_{AC} U^\dagger \approx \omega_c a^\dagger a + \xi e^{-i\Omega t} a^{\dagger 2} + \xi^* e^{i\Omega t} a^2 \\ &+ \sum_{i=1}^2 \left[\frac{\tilde{\omega}_a}{2} + \frac{4|\epsilon^{(i)}|^2}{\Delta^2} (\xi e^{-i\Omega t} a^{\dagger 2} + \text{h.c.}) + \frac{4|\epsilon^{(i)}|^2}{\Delta} a^\dagger a \right] \sigma_z^{(i)} \\ &+ \sum_{i=1}^2 \left[\left(\frac{2\epsilon^{(i)} \xi e^{-i\Omega t}}{\Delta} a^\dagger + \frac{|\epsilon^{(i)}|^2 \xi e^{-i\Omega t}}{\Delta^2} \right) \sigma_+^{(i)} + \text{h.c.} \right] \\ &+ (\mu_1 e^{-i(\Omega t + \phi_1)} \sigma_+^{(1)} \sigma_+^{(2)} + \text{h.c.}) + (\mu_2 e^{-i\phi_2} \sigma_+^{(1)} \sigma_-^{(2)} + \text{h.c.}), \end{aligned}$$

where h.c. refers to the Hermitian conjugate;

$$\tilde{\omega}_a = \omega_a + \frac{2(|\epsilon^{(1)}|^2 + |\epsilon^{(2)}|^2)}{\Delta}, \quad \mu_1 e^{-i\phi_1} = \frac{2\xi \epsilon^{(1)} \epsilon^{(2)}}{\Delta^2}, \quad \mu_2 e^{-i\phi_2} = \frac{\epsilon^{(1)} \epsilon^{(2)*}}{\Delta}.$$

Further, by adiabatically eliminating the degrees of freedom of the cavity mode, the following reduced two-atom Hamiltonian can be obtained:

$$H_A = \frac{\omega_a}{2} \sum_{i=1}^2 \sigma_z^{(i)} + (\mu_1 e^{-i(\Omega t + \phi_1)} \sigma_+^{(1)} \sigma_+^{(2)} + \text{h.c.}) + (\mu_2 e^{-i\phi_2} \sigma_+^{(1)} \sigma_-^{(2)} + \text{h.c.}),$$

where the terms of individual atomic interaction with the cavity are omitted due to the fact that

$$\frac{\omega_a}{2} \gg |\epsilon^{(i)}|^2 / \Delta, \quad |\xi \epsilon^{(i)}| / \Delta$$

under the large detuning condition $\Delta \gg \epsilon^{(i)}$. Since the parameter amplification coefficient ξ and the frequency Ω are tunable parameters, we have two control parameters μ_1 and Ω in H_A . In the interaction picture, H_A can be expressed as

$$H_A^{\text{eff}} = (\mu_1 e^{-i\phi_1} \sigma_+^{(1)} \sigma_+^{(2)} + \mu_1 e^{i\phi_1} \sigma_-^{(1)} \sigma_-^{(2)}) + (\mu_2 e^{-i\phi_2} \sigma_+^{(1)} \sigma_-^{(2)} + \mu_2 e^{i\phi_2} \sigma_-^{(1)} \sigma_+^{(2)}), \quad (3)$$

when the parameter Ω is fixed to be $2\omega_a$.

Besides the cavity mode, the atoms also interact with other modes in the environment, which leads to the atomic spontaneous emissions. In the case that the environmental modes are at the vacuum state, the dynamics of atoms can be described by the following master equation [18, 25]:

$$\dot{\rho} = -i[H_A^{\text{eff}} + H_{12}, \rho] + \sum_{i,j=1}^2 \Gamma_{ij} \left(\sigma_-^{(i)} \rho \sigma_+^{(j)} - \frac{1}{2} \rho \sigma_+^{(j)} \sigma_-^{(i)} - \frac{1}{2} \sigma_+^{(j)} \sigma_-^{(i)} \rho \right). \quad (4)$$

The parameters

$$\Gamma_{11} = \Gamma_{22} = \Gamma = \frac{\omega_a^3 \mu^2}{3\pi \epsilon_0 c^3} \quad (5)$$

are the spontaneous emission rates of the individual atoms, where $\mu = |\vec{\mu}|$ is the magnitude of the transition dipole momentum, while

$$\Gamma_{12} = \Gamma_{21} = \Gamma F(k_0 r_{12}) \quad (6)$$

represent the collective spontaneous emission rates induced by the coupling between the atoms. The function $F(k_0 r_{12})$ can be expressed as [18, 25]

$$F(k_0 r_{12}) = \frac{3}{2} \left\{ (1 - 3 \cos^2 \theta) \left[\frac{\cos(k_0 r_{12})}{(k_0 r_{12})^2} - \frac{\sin(k_0 r_{12})}{(k_0 r_{12})^3} \right] + \sin^2 \theta \frac{\sin(k_0 r_{12})}{k_0 r_{12}} \right\},$$

where $k_0 = \omega_a/c$, and θ is the angle between the dipole moment vector $\vec{\mu}$ and the vector $\vec{r}_{12} = \vec{r}^{(1)} - \vec{r}^{(2)}$; $r_{12} = |\vec{r}_{12}|$ is the distance between the two atoms. The spontaneous emission process also introduces an additional coherent dipole–dipole interaction between the atoms:

$$H_{12} = \eta (\sigma_+^{(1)} \sigma_-^{(2)} + \sigma_-^{(1)} \sigma_+^{(2)}),$$

where the coefficient η in H_{12} can be written as [25]

$$\eta = \frac{3}{4} \Gamma \left\{ [1 - 3 \cos^2 \theta] \left[\frac{\sin(k_0 r_{12})}{(k_0 r_{12})^2} + \frac{\cos(k_0 r_{12})}{(k_0 r_{12})^3} \right] - \sin^2 \theta \frac{\cos(k_0 r_{12})}{k_0 r_{12}} \right\}. \quad (7)$$

3. Independent amplitude damping decoherence channel

When the distance r_{12} between the two atoms is far greater than the resonant wavelength $1/k_0 = c/\omega_a$ of the atom, i.e., $k_0 r_{12} \rightarrow \infty$, the amplitude damping decoherence of the two atoms can be taken independently. Consequently, from equations (6) and (7), we have $\eta, \Gamma_{12}, \Gamma_{21} \rightarrow 0$, from which the following master equation holds:

$$\dot{\rho} = -i[H_A^{\text{eff}}, \rho] + \Gamma \mathbb{D}[\sigma_-^{(1)}] \rho + \Gamma \mathbb{D}[\sigma_-^{(2)}] \rho, \quad (8)$$

where the superoperator $\mathbb{D}[L]\rho$ is defined as

$$\mathbb{D}[L]\rho = L\rho L^\dagger - \frac{1}{2} L^\dagger L\rho - \frac{1}{2} \rho L^\dagger L,$$

and the two Lindblad terms $\mathbb{D}[\sigma_-^{(1)}]\rho, \mathbb{D}[\sigma_-^{(2)}]\rho$ represent the amplitude damping decoherence channels acting on the two atoms with the damping rate $\Gamma > 0$.

To measure the quantum entanglement, we use the concurrence [3] between the two atoms of the quantum state ρ :

$$C(\rho) = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}, \quad (9)$$

where λ_i 's are the square roots of the eigenvalues, in a decreasing order, of the matrix,

$$M = \rho (\sigma_y^{(1)} \sigma_y^{(2)}) \rho^* (\sigma_y^{(1)} \sigma_y^{(2)}),$$

and ρ^* is the complex conjugate of ρ .

It is known that, in the absence of the squeezed field, a two-atom system will always be disentangled under independent amplitude damping decoherence channels (see, e.g., [23]), and this is not recoverable by any local operations. However, the entanglement can be partially protected via the intermediate squeezed field, because the solution $\rho(t)$ of equation (8) tends to a stationary state,

$$\rho_\infty = \frac{2\mu_1\Gamma}{4\mu_1^2 + \Gamma^2} \rho_m + \left(1 - \frac{2\mu_1\Gamma}{4\mu_1^2 + \Gamma^2}\right) \rho_s, \quad (10)$$

as a convex combination of a pure maximally entangled state

$$\rho_m(\phi_1) = \frac{1}{2} \begin{pmatrix} 1 & & e^{-i(\phi_1 - \frac{\pi}{2})} \\ & 0 & \\ e^{i(\phi_1 - \frac{\pi}{2})} & & 1 \end{pmatrix} \quad (11)$$

and a diagonal separable state

$$\rho_s = \text{diag}(1 - 3\beta, \beta, \beta, \beta),$$

where

$$\beta = \frac{1}{8} \left(1 - \sqrt{1 - \left(\frac{4\mu_1\Gamma}{4\mu_1^2 + \Gamma^2}\right)^2}\right).$$

The subscript ‘ m ’ is an abbreviation of ‘maximally entangled’, and the subscript ‘ s ’ refers to ‘separable’. The corresponding stationary concurrence is

$$C(\rho_\infty) = \max \left\{ \frac{2\mu_1(\Gamma - \mu_1)}{4\mu_1^2 + \Gamma^2}, 0 \right\}, \quad (12)$$

which, when the coupling strength μ_1 is tuned to be

$$\mu_1 = \frac{1}{\sqrt{5} + 1} \Gamma,$$

reaches its maximum value,

$$C_{\max} = \frac{\sqrt{5} - 1}{4} \approx 0.31 > 0.$$

The plot of $C(\rho_\infty)$ versus μ_1/Γ is shown in figure 2.

The cumbersome proof of equations (10) and (12) is shown in appendix A. We adopt here an ideal model in which the two atoms have precise positions $\vec{r}^{(i)}$. In real systems, position fluctuations are always presented, i.e.,

$$\tilde{\vec{r}}^{(i)} = \vec{r}^{(i)} + \delta\vec{r}^{(i)}, \quad i = 1, 2,$$

where $\tilde{\vec{r}}^{(i)}$ is the actual position of the i th atom and $\delta\vec{r}^{(i)}$ is the corresponding fluctuation. From equation (2), the actual coupling coefficients should be

$$\tilde{\mu}_1 e^{i\tilde{\phi}_1} = e^{i\vec{k} \cdot (\delta\vec{r}^{(1)} + \delta\vec{r}^{(2)})} \mu_1 e^{i\phi_1} = \mu_1 e^{i(\phi_1 + \vec{k} \cdot (\delta\vec{r}^{(1)} + \delta\vec{r}^{(2)}))},$$

which, consequently, fluctuates the phase by

$$\delta\phi_1 = \tilde{\phi}_1 - \phi_1 = \vec{k} \cdot (\delta\vec{r}^{(1)} + \delta\vec{r}^{(2)}) = \delta\phi^{(1)} + \delta\phi^{(2)} \quad (13)$$

for the pure maximally entangled state ρ_m (which now it should be $\rho_m(\tilde{\phi}_1) = \rho_m(\phi_1 + \delta\phi_1)$) due to the fluctuations of the positions of the atoms. Assume that $\delta\phi^{(1)}$ and $\delta\phi^{(2)}$ obey Gaussian

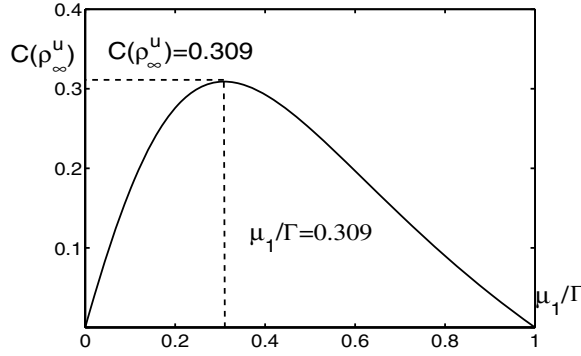


Figure 2. Plot of $C(\rho_\infty)$ versus μ_1/Γ .

distributions with means 0 and variances $2\gamma_1$ and $2\gamma_2$; one can verify by averaging over the random fluctuations that the pure maximally entangled state ρ_m is blurred into a mixed state:

$$\begin{aligned} \bar{\rho}_m &= \int_{-\infty}^{+\infty} \frac{d\delta\phi^{(1)}}{\sqrt{4\pi\gamma_1}} e^{-(\delta\phi^{(1)})^2/4\gamma_1} \int_{-\infty}^{+\infty} \frac{d\delta\phi^{(2)}}{\sqrt{4\pi\gamma_2}} e^{-(\delta\phi^{(2)})^2/4\gamma_2} \rho_m(\tilde{\phi}_1) \\ &= \frac{1}{2} \begin{pmatrix} 1 & & e^{-(\gamma_1+\gamma_2)} e^{-i(\phi_1-\frac{\pi}{2})} \\ & 0 & \\ e^{-(\gamma_1+\gamma_2)} e^{i(\phi_1-\frac{\pi}{2})} & & 1 \end{pmatrix}. \end{aligned}$$

Apparently, the resulting entanglement is also reduced. In fact, in this case, the stationary state should be

$$\bar{\rho}_\infty = \frac{2\mu_1\Gamma}{4\mu_1^2 + \Gamma^2} \bar{\rho}_m + \left(1 - \frac{2\mu_1\Gamma}{4\mu_1^2 + \Gamma^2}\right) \rho_s,$$

with a modified stationary concurrence

$$C(\bar{\rho}_\infty) = \left\{ e^{-(\gamma_1+\gamma_2)} \frac{2\mu_1\Gamma}{4\mu_1^2 + \Gamma^2} - \frac{2\mu_1^2}{4\mu_1^2 + \Gamma^2}, 0 \right\}.$$

The corresponding maximum stationary concurrence can be further calculated as

$$\bar{C}_{\max} = \frac{1}{4} (\sqrt{4e^{-(\gamma_1+\gamma_2)} + 1} - 1).$$

Obviously, we have $\bar{C}_{\max} > 0$, which means that our strategy is still valid compared with the case without the squeezed field.

However, the maximum stationary concurrence is reduced by the dephasing effects caused by the fluctuations of the positions of the atoms. In order to estimate the influence of the fluctuations on the stationary entanglement, it can be estimated from equation (12) that

$$\gamma_i = \frac{1}{2} \text{var}(\delta\phi^{(i)}) \leq \frac{1}{2} |\vec{k}|^2 \text{var}(|\delta\vec{r}^{(i)}|) = 2\pi^2 \left(\frac{\delta r^{(i)}}{\lambda}\right)^2,$$

where λ is the wavelength of the field in the cavity, $\text{var}(\delta\phi^{(i)})$ is the variance of $\delta\phi^{(i)}$, and $\delta r^{(i)} = \sqrt{\text{var}(|\delta\vec{r}^{(i)}|)}$ represents the magnitude of the position fluctuation for the i th atom. Therefore, if one is capable of trapping the atom in the cavity such that

$$\delta r^{(i)} \ll \lambda, \tag{14}$$

the dephasing coefficients γ_i can be neglected. This is possible under the present atom trapping and cooling technique since the wavelength λ is of the order of μm (see, e.g., [29–32]). In this case, the perturbed maximum stationary concurrence \bar{C}_{\max} is deviated slightly from the ideal maximum stationary concurrence C_{\max} (e.g., $\bar{C}_{\max}/C_{\max} \approx 90\%$ when $\delta r^{(i)}/\lambda = 0.05$ as assumed in [29]). We can also see the influence from the following example with parameters given in [30], in which the mass m of the atom (the Cs atom), the oscillating frequency ω of the external freedom of the atom (which is different from ω_a), the effective temperature T_{eff} of the atom and the wavelength λ of the field in the cavity are given as

$$\begin{aligned} m &= 133 \times 1.66 \times 10^{-27} \text{ kg} \approx 2.2 \times 10^{-25} \text{ kg}, \\ \omega &= 2\pi \times 0.53 \times 10^6 \text{ Hz} \approx 3.3 \times 10^6 \text{ Hz}, \\ T_{\text{eff}} &= 1.3 \times 10^{-4} \text{ K}, \quad \lambda \approx 0.9 \times 10^{-6} \text{ m}. \end{aligned}$$

Here, we choose an effective temperature T_{eff} of the atom that is ten times greater than the lowest cooling temperature ($13 \mu\text{K}$) given in [30], under which the position of the atom can be taken as a classical parameter because

$$\hbar\omega \ll \frac{1}{2}k_B T_{\text{eff}},$$

where k_B is the Boltzmann constant. In this case, the position fluctuation of the atom can be estimated from

$$\frac{1}{2}m\omega^2(\delta r^{(i)})^2 = \frac{1}{2}k_B T_{\text{eff}},$$

which leads to

$$\delta r^{(i)} = \sqrt{\frac{k_B T_{\text{eff}}}{m\omega^2}} = \sqrt{\frac{1.38 \times 10^{-23} \times 1.3 \times 10^{-4}}{2.2 \times 10^{-25} \times 3.3^2 \times 10^{12}}} \approx 2.7 \times 10^{-8} \text{ m}.$$

Thus, we have

$$\frac{\delta r^{(i)}}{\lambda} \approx \frac{2.8 \times 10^{-8}}{0.9 \times 10^{-6}} = 0.03,$$

from which it can be calculated that $\bar{C}_{\max}/C_{\max} \approx 97\%$.

Although the stationary concurrence may not be strong enough to be directly applied in quantum information processing and would be deteriorated by dephasing effects caused by noises such as the position fluctuations of the atoms, it is still hopeful to be used for entanglement protection. In fact, the fidelity between the stationary state $\bar{\rho}_\infty$ and the maximally entangled state ρ_m can be calculated as

$$F(\bar{\rho}_\infty) = \text{tr}(\bar{\rho}_\infty \rho_m) = \frac{e^{-(\gamma_1+\gamma_2)}\mu_1\Gamma - \mu_1^2}{4\mu_1^2 + \Gamma^2} + \frac{1}{2}.$$

Optimally, it should be

$$\bar{F}_{\max} = \frac{1}{8}(\sqrt{4e^{-(\gamma_1+\gamma_2)} + 1} - 1) + \frac{1}{2} > \frac{1}{2}. \tag{15}$$

Since the maximum fidelity \bar{F}_{\max} is always larger than 0.5, we can, in principle, increase the stationary entanglement by introducing an additional entanglement purification process [33, 34].

To illustrate our proposal, let us discuss their applications in some typical circumstances. Firstly, consider the initial states ρ_0^1 at the maximally entangled state

$$\rho_0^1 = \frac{1}{2} \begin{pmatrix} 1 & & 1 \\ & 0 & \\ 1 & & 1 \end{pmatrix},$$

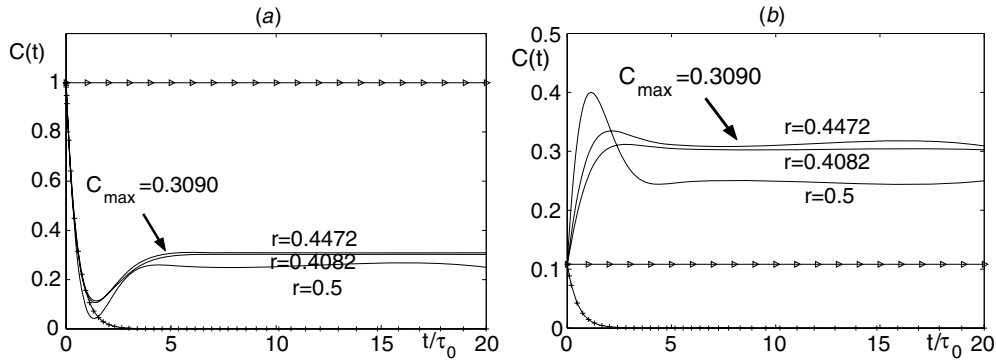


Figure 3. Plots of concurrence $C(t)$ where the initial states are chosen as (a) the maximally entangled state ρ_0^1 and (b) the mixed entangled state ρ_0^2 . The plus-sign lines denote the uncontrolled trajectories, the triangle lines are for the free trajectories in the absence of control and decoherence, and the solid lines represent the controlled trajectories with chosen parameters $r = 2\mu_1\Gamma/(4\mu_1^2 + \Gamma^2)$, which are the proportion of the maximally entangled state ρ_m in equation (10).

and ρ_0^2 at the mixed entangled state

$$\rho_0^2 = \begin{pmatrix} 0.85 & & & 0.1 \\ & 0.03 & & \\ & & 0.07 & \\ 0.1 & & & 0.05 \end{pmatrix}.$$

Moreover, let $\Gamma = 1/\tau_0$, where τ_0 is the relaxing time constant. Simulation results are shown in figure 3.

It is shown in figure 3 that the entanglement of the quantum states always decays to zero without control as what is known in the literature [23]. The corresponding stationary state is the two-atom ground state:

$$\rho_\infty^u = |00\rangle\langle 00| = \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix},$$

which is at the boundary of the set of all separable states. The superscript ‘ u ’ refers to the ‘uncontrolled’ system. When our strategy is applied, the entanglement can be remarkably retrieved against decoherence, and the maximum concurrence of the stationary state is

$$C_{\max} = \frac{\sqrt{5} - 1}{4} \approx 0.31,$$

when

$$r = \frac{2\mu_1\Gamma}{4\mu_1^2 + \Gamma^2} = \frac{1}{\sqrt{5}} \approx 0.45.$$

It is also noted that our strategy can enhance the entanglement of the stationary state of the naked atoms (i.e., neither control nor decoherence exists).

4. Collective amplitude damping decoherence channel

When the distance between the atoms is far shorter than the resonant wavelength of the atom, i.e., $k_0 r_{12} \rightarrow 0$, from equations (6) and (7) we have

$$\eta \rightarrow \eta_0 = \frac{3\Gamma}{4k_0^3 r_{12}^3} (1 - 3 \cos^2 \theta), \quad \Gamma_{12} = \Gamma_{21} \rightarrow \Gamma,$$

which corresponds to a two-atom collective amplitude damping decoherence channel [35]. In this case, the master equation of the two atoms becomes

$$\dot{\rho} = -i[H_A^{\text{eff}} + \eta_0(\sigma_+^{(1)}\sigma_-^{(2)} + \sigma_-^{(1)}\sigma_+^{(2)}), \rho] + \Gamma\mathbb{D}[S_-]\rho, \quad (16)$$

where the two-atom operator $S_- = \sigma_-^{(1)} + \sigma_-^{(2)}$, and $\Gamma > 0$ is the damping rate. Because the two atoms are very close to each other, from equation (2) the coupling strength between each atom and the cavity can be taken as identical, i.e., $\epsilon^{(1)} = \epsilon^{(2)} = \epsilon$, so that the interaction Hamiltonian H_A^{eff} can be expressed as

$$H_A^{\text{eff}} = \mu_1(e^{-i\phi_1}\sigma_+^{(1)}\sigma_+^{(2)} + e^{i\phi_1}\sigma_-^{(1)}\sigma_-^{(2)}) + \mu_2(\sigma_+^{(1)}\sigma_-^{(2)} + \sigma_-^{(1)}\sigma_+^{(2)}),$$

where

$$\mu_1 e^{-i\phi_1} = 2\xi\epsilon^2/\Delta^2, \quad \mu_2 = |\epsilon|^2/\Delta.$$

In the absence of the intermediate squeezed field, i.e., $\mu_1 = \mu_2 = 0$, the stationary state of the two-atom system,

$$\rho_\infty^u = (1 - \kappa)\tilde{\rho}_m + \kappa\rho_0, \quad (17)$$

is a convex combination of the maximally entangled state

$$\tilde{\rho}_m = \frac{1}{2} \begin{pmatrix} 0 & & & \\ & 1 & -1 & \\ & -1 & 1 & \\ & & & 0 \end{pmatrix}, \quad (18)$$

and the two-atom ground state

$$\rho_0 = \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix},$$

where the weight $\kappa \in [0, 1]$ is determined by the initial density matrix:

$$\kappa = \text{tr} \left[\left(\frac{1}{4} \sigma_z^{(1)} \sigma_z^{(2)} \right) \rho(t_0) \right] + \frac{\sqrt{2}}{2} \text{tr}(\Omega_{23}^x \rho(t_0)) + \frac{3}{4},$$

and

$$\Omega_{23}^x = \frac{1}{\sqrt{2}} \begin{pmatrix} & & & 0 \\ & & 1 & \\ & 1 & & \\ 0 & & & \end{pmatrix}.$$

The resulting stationary concurrence is

$$C(\rho_\infty^u) = 1 - \kappa.$$

When the intermediate squeezed field is presented, the corresponding two-atom stationary state

$$\rho_\infty = s\tilde{\rho}_m + r\rho_m + (1-s-r)\tilde{\rho}_s \quad (19)$$

is a convex combination of the maximally entangled states $\tilde{\rho}_m$ and ρ_m given in equations (18) and (11) respectively, and a diagonal separable state

$$\tilde{\rho}_s = \text{diag}(\tilde{\beta}_1 + \tilde{\beta}_2, \frac{1}{2} - \tilde{\beta}_2, \frac{1}{2} - \tilde{\beta}_2, \tilde{\beta}_2 - \tilde{\beta}_1),$$

where

$$\tilde{\beta}_1 = \frac{\kappa\Gamma^2}{2(\Gamma^2 + 3\mu_1^2)}, \quad \tilde{\beta}_2 = \frac{\kappa(\Gamma^2 + 2\mu_1^2)}{2(\Gamma^2 + 3\mu_1^2)}.$$

The weights s and r are, respectively,

$$s = 1 - \frac{7}{6}\kappa + \frac{\Gamma^2 - 3\mu_1^2}{\Gamma^2 + 3\mu_1^2}\kappa, \quad r = \frac{2\Gamma\mu_1\kappa}{\Gamma^2 + 3\mu_1^2}.$$

It can be examined that when the parameter μ_1 is in the range

$$\frac{\kappa - \sqrt{-9\kappa^2 + 22\kappa - 12}}{6 - 5\kappa} \leq \frac{\mu_1}{\Gamma} \leq \frac{\kappa + \sqrt{-9\kappa^2 + 22\kappa - 12}}{6 - 5\kappa}, \quad (20)$$

the resulting stationary concurrence $C(\rho_\infty)$ is superior to $C(\rho_\infty^u)$ without the intermediate squeezed field:

$$C(\rho_\infty) = \frac{(\Gamma^2 + 2\mu_1^2 + 2\Gamma\mu_1)\kappa}{\Gamma^2 + 3\mu_1^2} - 1 \geq C(\rho_\infty^u) = 1 - \kappa. \quad (21)$$

The interval given in (19) is nonempty if and only if

$$\frac{11}{9} - \frac{1}{9}\sqrt{13} \leq \kappa \leq 1, \quad (22)$$

otherwise, our strategy is not capable of improving the stationary concurrence. Moreover, the maxima of $C(\rho_\infty)$ is achieved when

$$\mu_1 = \frac{2}{\sqrt{13} + 1}\Gamma,$$

and the corresponding maximum value is

$$C_{\max} = \frac{\sqrt{13} + 5}{6}\kappa - 1.$$

The plots of the stationary concurrence versus the coupling strength for different κ are shown in figure 4. In figure 4(a), the controlled stationary concurrence is superior to the uncontrolled one only in the interval given by equation (19), while, in figure 4(b), the controlled stationary concurrence is always better than the uncontrolled one.

The fact that our strategy is effective only when the parameter κ is sufficiently large comes from the competition between $\tilde{\rho}_m$ and ρ_m in equation (19), where $\tilde{\rho}_m$ comes from the dissipation effect and ρ_m is induced by our proposal. When κ is close to 0, the dissipation dominates and hence the control fails, while, when κ is close to 1, the control becomes effective.

As has been indicated in section 3, the fluctuations of the positions of the atoms would bring an uncertain phase shift for the maximally entangled state ρ_m , which may deteriorate the stationary concurrence. The calculations are like those in section 3, so we omit them here.

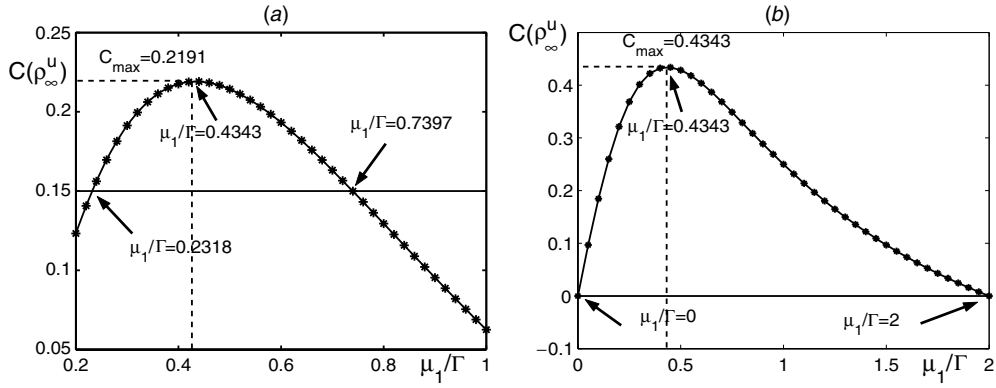


Figure 4. Plots of $C(\rho_\infty)$ versus μ_1/Γ for (a) $\kappa = 0.85$, (b) $\kappa = 1$. The asterisk line is for the controlled stationary concurrence, and the solid line is for the uncontrolled stationary concurrence.

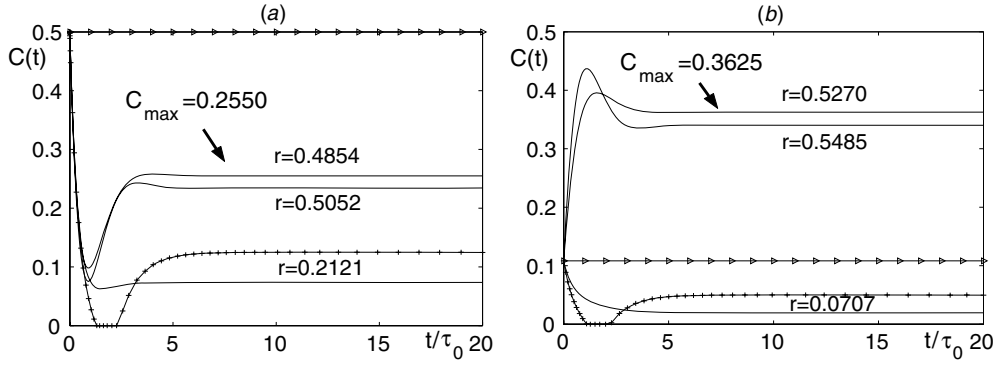


Figure 5. Plots of the concurrence $C(t)$ where the system state is initialized for (a) ρ_0^1 and (b) ρ_0^2 . The plus-sign lines denote the uncontrolled trajectories, the triangle lines are for the free trajectories in the absence of control and decoherence, and the solid lines represent the controlled trajectories with different parameters $r = 2\Gamma\mu_1\kappa/(\Gamma^2 + 3\mu_1^2)$, which are the proportion of the squeezed field induced maximally entangled state ρ_m in equation (19).

Figure 5 shows some numerical examples, where the initial states are, respectively,

$$\rho_0^1 = \begin{pmatrix} 3/8 & & 3/8 \\ & 1/8 & \\ 3/8 & & 3/8 \end{pmatrix},$$

and

$$\rho_0^2 = \begin{pmatrix} 0.85 & & 0.1 \\ & 0.03 & \\ 0.1 & & 0.05 \end{pmatrix}.$$

The simulation results show that the stationary state of the uncontrolled system under the collective amplitude damping decoherence may remain entangled which is quite different

from the independent decoherence channel, and this feature has been utilized in the literature [15–17] to create entanglement between qubits. Our strategy may further increase the entanglement in the stationary entangled state, as shown in comparison between the plus-sign lines and the solid lines.

Another feature of the collective amplitude damping decoherence channel observed from figure 5 is that the maximum concurrence depends on the initial state. For certain values of r , our control strategy may have worse performance than that induced by the natural dissipation. Both figures 5(a) and (b) provide such a case where the solid line (controlled trajectory) goes below the plus-sign line (uncontrolled trajectory). The corresponding control parameter μ_1 is outside the interval given in equation (19).

5. Mixed amplitude damping decoherence channel

In actual experiments, the decoherence channel is never perfectly collective, because it is hard to place the two atoms in a cavity close enough. The existing atom trapping and cooling techniques [36, 37] can only hold two atoms approximately at the distance of the same order of the resonant wavelength of the atom. Thus, it is more realistic to treat the resulting decoherence channel as a mixture of an independent amplitude damping decoherence channel and a collective amplitude damping decoherence channel, as shown in the following master equation:

$$\begin{aligned} \dot{\rho} = & -i[H_A^{\text{eff}}, \rho] + \sum_{i=1}^2 \Gamma \mathcal{D}[\sigma_-^{(i)}] \rho + \Gamma_{12} \left(\sigma_-^{(1)} \rho \sigma_+^{(2)} - \frac{1}{2} \{ \sigma_+^{(2)} \sigma_-^{(1)}, \rho \} \right) \\ & + \Gamma_{12} \left(\sigma_-^{(2)} \rho \sigma_+^{(1)} - \frac{1}{2} \{ \sigma_+^{(1)} \sigma_-^{(2)}, \rho \} \right), \end{aligned} \quad (23)$$

where $0 < \Gamma_{12} < \Gamma$.

It can be verified that the stationary state of the uncontrolled system is nothing but the separable two-atom ground state $\rho_\infty^u = |00\rangle\langle 00|$ in which entanglement completely disappears, as well as in the case of the independent decoherence channel. By introducing the intermediate squeezed field, we can stabilize the system at the same stationary state given in equation (10).

6. Conclusions

In summary, we proposed a two-atom entanglement control strategy, via a controllable squeezed field coupled to the two atoms, to protect entanglement from the spontaneous emission process. The parameter amplification coefficient of the squeezed field can be tuned to generate a non-local Hamiltonian, which can be used to maintain entanglement of the two-atom states against decoherence. For the independent amplitude damping decoherence channel, we can partially recover the entanglement of the quantum state which otherwise will be completely lost. For the collective amplitude damping decoherence channel, our strategy can effectively enhance the entanglement of the stationary state compared with the dissipation-induced strategies provided that the uncontrolled stationary state is not tightly entangled.

The proposed entanglement control strategy is an open-loop control strategy, where no measurements are done during the course of control. Such control strategies require exact values of the system parameters, and can badly suffer from the uncertainty of these parameters, which may bring remarkable derivation of the stationary concurrence from the ideal values. This problem is hopefully solvable by quantum feedback controls.

Another direction of the succeeding research will be the application in solid state systems. In such systems, controllable coupling between qubits is easier to be achieved [38, 39] compared with the optical systems. However, interactions between the solid-state systems and their environments are more complicated, which may lead to non-Markovian noises [40]. To what extent the controllable non-local unitary operations can preserve entanglement against non-Markovian noises is an interesting problem to be explored, for which existing decoherence suppression strategies [41–43] may be helpful.

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Appendix A. Proof of equations (10) and (12)

Firstly, we transform the control model (8) from the complex matrix space into the real vector space, i.e., the so-called coherence vector picture [44–47]. With respect to the inner product $\langle X, Y \rangle = \text{tr}(X^\dagger Y)$, we define the following orthonormal basis for all two-atom operators:

$$\left\{ \frac{1}{2} I_{4 \times 4}, \Omega_{14}^x, \Omega_{14}^y, \Omega_{23}^x, \Omega_{23}^y, \frac{1}{2} \sigma_x^{(1)}, \frac{1}{2} \sigma_y^{(1)}, \frac{1}{2} \sigma_x^{(2)}, \frac{1}{2} \sigma_y^{(2)}, \frac{1}{2} \sigma_x^{(1)} \sigma_z^{(2)}, \right. \\ \left. \times \frac{1}{2} \sigma_z^{(1)} \sigma_x^{(2)}, \frac{1}{2} \sigma_y^{(1)} \sigma_z^{(2)}, \frac{1}{2} \sigma_z^{(1)} \sigma_y^{(2)}, \Omega_{14}^z, \Omega_{23}^z, \frac{1}{2} \sigma_z^{(1)} \sigma_z^{(2)} \right\}, \quad (\text{A.1})$$

where

$$\Omega_{14}^x = \frac{1}{\sqrt{2}} \begin{pmatrix} & & 1 \\ & 0 & \\ 1 & & \end{pmatrix}, \quad \Omega_{14}^y = \frac{1}{\sqrt{2}} \begin{pmatrix} & & -i \\ & 0 & \\ i & & 0 \end{pmatrix}, \\ \Omega_{23}^x = \frac{1}{\sqrt{2}} \begin{pmatrix} & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix}, \quad \Omega_{23}^y = \frac{1}{\sqrt{2}} \begin{pmatrix} & & 0 \\ & i & -i \\ 0 & & \end{pmatrix}, \\ \Omega_{14}^z = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \\ & & & -1 \end{pmatrix}, \quad \Omega_{23}^z = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & & \\ & 1 & \\ & & -1 \\ & & & 0 \end{pmatrix}.$$

Under this basis, the system density matrix can be expressed as

$$\rho = \frac{1}{4} I_{4 \times 4} + \sum_{i=1}^{15} m_i \Omega_i,$$

where $m_i = \text{tr}(\Omega_i \rho)$ and $\Omega_i, i = 1, \dots, 15$, are the basis matrices in equation (23) except $\frac{1}{2} I_{4 \times 4}$. $m = (m_1, \dots, m_{15})^T$ is called the coherence vector of ρ .

In the coherence vector picture, the master equation (8) can be rewritten as [44–47]

$$\dot{m} = O_A m + D m + g, \quad (\text{A.2})$$

where the orthogonal matrix O_A is the adjoint representation [45] of $-iH_A^{\text{eff}}$. The affine term ‘ $Dm + g$ ’ is that of the Lindblad terms:

$$\Gamma \mathbb{D}[\sigma_-^{(1)}] \rho + \Gamma \mathbb{D}[\sigma_-^{(2)}] \rho,$$

where $D \leq 0$, and g is a constant vector. Further, divide m into the following sub-vectors:

$$\begin{aligned} m^p &= (m_{14}^x, m_{14}^y, m_{23}^x, m_{23}^y)^T, \\ m^\eta &= (m_{14}^z, m_{23}^z, m_{zz})^T, \\ m^\epsilon &= (m_{x0}, m_{y0}, m_{0x}, m_{0y}, m_{xz}, m_{zx}, m_{yz}, m_{zy})^T, \end{aligned} \tag{A.3}$$

where

$$m_{14}^\alpha = \text{tr}(\Omega_{14}^\alpha \rho), \quad m_{23}^\beta = \text{tr}(\Omega_{23}^\beta \rho), \quad \alpha, \beta = x, y, z,$$

$$m_{ij} = \text{tr}[(\frac{1}{2}\sigma_i^{(1)}\sigma_j^{(2)})\rho], \quad i, j = 0, x, y, z,$$

and $\sigma_0^{(i)} = I_{2 \times 2}$, $i = 1, 2$ are the 2×2 identity operators acting on the i th atom. Then, (A.4) can be grouped into

$$\begin{aligned} \dot{m}^p &= \sum_{i=1}^4 u_i O_i^\eta m^\eta + D^p m^p, \\ \dot{m}^\eta &= \sum_{i=1}^4 u_i (-O_i^{\eta T}) m^p + D^\eta m^\eta + g^\eta, \\ \dot{m}^\epsilon &= \sum_{i=1}^4 u_i O_i^\epsilon m^\epsilon + D^\epsilon m^\epsilon, \end{aligned} \tag{A.4}$$

where

$$\begin{aligned} u_1 &= 8\mu_1 \cos \phi_1, & u_2 &= 8\mu_1 \sin \phi_1, & u_3 &= 8\mu_2 \cos \phi_2, & u_4 &= -8\mu_2 \sin \phi_2, \\ D^p &= -4\Gamma \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}, & O_1^\eta &= \begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ O_2^\eta &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & O_3^\eta &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, & O_4^\eta &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ D^\eta &= -4\Gamma \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\sqrt{2} & 0 & 2 \end{pmatrix}, & g^\eta &= \begin{pmatrix} 2\sqrt{2}\Gamma \\ 0 \\ 0 \end{pmatrix}. \end{aligned} \tag{A.5}$$

O_i^ϵ are all skew-symmetric matrices and $D^\epsilon < 0$, thereby

$$\frac{d}{dt}((m^\epsilon)^T m^\epsilon) = (m^\epsilon)^T D^\epsilon m^\epsilon < 0, \quad \forall m^\epsilon \neq 0,$$

which implies that $m^\epsilon \rightarrow 0$ when $t \rightarrow \infty$.

With simple calculations, the following stationary solution can be obtained for equation (A.4):

$$\begin{aligned} m^\epsilon &= 0, & m_{23}^x(\infty) &= m_{23}^y(\infty) = m_{23}^z(\infty) = 0, \\ m_{14}^x(\infty) &= \frac{\sqrt{2}\mu_1\Gamma}{4\mu_1^2 + \Gamma^2} \cos\left(\phi_1 - \frac{\pi}{2}\right), \\ m_{14}^y(\infty) &= \frac{\sqrt{2}\mu_1\Gamma}{4\mu_1^2 + \Gamma^2} \sin\left(\phi_1 - \frac{\pi}{2}\right), \\ m_{14}^z(\infty) &= \sqrt{2}m_{zz}(\infty) = \frac{\sqrt{2}\Gamma^2}{8\mu_1^2 + 2\Gamma^2}, \end{aligned}$$

from which we can obtain the corresponding decomposition (10) of the stationary state ρ_∞ .

Going back to the density matrix, one can find that the stationary state has the following form:

$$\rho_\infty = \begin{pmatrix} a & & & w \\ & b & z & \\ & z^* & c & \\ w^* & & & d \end{pmatrix}, \tag{A.6}$$

whose concurrence can be analytically solved to be [19–21]

$$C(\rho_\infty) = 2 \max\{|w| - \sqrt{bc}, |z| - \sqrt{ad}, 0\}. \tag{A.7}$$

The above equation leads to the stationary concurrence $C(\rho_\infty)$ given in equation (12).

Appendix B. Proof of equations (17)–(21)

Similar to what we have done in appendix A, the controlled master equation (16) can be grouped into the following control equations in the coherence vector picture:

$$\begin{aligned} \dot{m}_{14}^x &= 8\mu_1 \sin\theta_1 m_{14}^z - 4\Gamma m_{14}^x, \\ \dot{m}_{14}^y &= -8\mu_1 \cos\theta_1 m_{14}^z - 4\Gamma m_{14}^y, \\ \dot{m}_{14}^z &= 8\mu_1 \cos\theta_1 m_{14}^y - 8\mu_1 \sin\theta_1 m_{14}^x - 4\Gamma m_{14}^z + 2\sqrt{2}\Gamma + 4\Gamma m_{23}^x, \\ \dot{m}_{23}^x &= -4\Gamma m_{23}^x - 4\Gamma m_{14}^z + 4\sqrt{2}\Gamma m_{zz}, \\ \dot{m}_{23}^y &= -8(\mu_2 + \eta_0)m_{23}^z - 4\Gamma m_{23}^y, \\ \dot{m}_{23}^z &= 8(\mu_2 + \eta_0)m_{23}^y - 4\Gamma m_{23}^z, \\ \dot{m}_{zz} &= 4\sqrt{2}\Gamma m_{14}^z - 8\Gamma m_{zz} + 4\sqrt{2}\Gamma m_{23}^x. \end{aligned} \tag{B.1}$$

As well as the independent amplitude damping decoherence model, the sub-vector $m^\epsilon = (m_{x0}, \dots, m_{zy})^T$ always goes to zero when $t \rightarrow \infty$, and it will not affect $C(\rho_\infty)$, so we will not discuss $m^\epsilon(t)$ here. The fourth and the last equations in equation (B.1) imply a conservation law:

$$m_{zz}(t) + \sqrt{2}m_{23}^x(t) \equiv m_{zz}(t_0) + \sqrt{2}m_{23}^x(t_0) \triangleq 2\kappa - \frac{3}{2}.$$

Substituting $\mu_1 = \mu_2 = 0$ into equation (B.1), we have the following uncontrolled stationary solution:

$$\begin{aligned} m_{14}^x(\infty) &= m_{14}^y(\infty) = m_{23}^y(\infty) = m_{23}^z(\infty) = 0, \\ m_{14}^z(\infty) &= \frac{\sqrt{2}}{2}\kappa, & m_{23}^x(\infty) &= \frac{\sqrt{2}}{2}(\kappa - 1), & m_{zz}(\infty) &= \kappa - \frac{1}{2}, \end{aligned}$$

from which the decomposition (17) can be obtained and $C(\rho_\infty^u) = 1 - \kappa$.

Further, we can obtain the controlled stationary solution of equation (B.1):

$$m_{14}^x(\infty) = \frac{\sqrt{2}\mu_1\Gamma\kappa}{\Gamma^2 + 3\mu_1^2} \cos\left(\phi_1 - \frac{\pi}{2}\right), \quad m_{14}^y(\infty) = \frac{\sqrt{2}\mu_1\Gamma\kappa}{\Gamma^2 + 3\mu_1^2} \sin\left(\phi_1 - \frac{\pi}{2}\right),$$

$$m_{23}^y(\infty) = m_{23}^z(\infty) = 0, \quad m_{14}^z(\infty) = \frac{\sqrt{2}\kappa\Gamma^2}{2(\Gamma^2 + 3\mu_1^2)},$$

$$m_{zz}(\infty) = \kappa - \frac{1}{2} - \frac{\mu_1^2\kappa}{\Gamma^2 + 3\mu_1^2}, \quad m_{23}^x(\infty) = \frac{\sqrt{2}}{2} \left(\kappa - 1 + \frac{\mu_1^2\kappa}{\Gamma^2 + 3\mu_1^2} \right),$$

which leads to the decomposition (19). Further, it can be calculated that

$$C(\rho_\infty) = \max\{F_1(\mu_1), F_2(\mu_2), 0\},$$

where

$$F_1(\mu_1) = -\kappa + 1 + \frac{\mu_1^2}{\Gamma^2 + 3\mu_1^2}\kappa - \frac{2\mu_1\sqrt{\Gamma^2 + \mu_1^2}}{\Gamma^2 + 3\mu_1^2}\kappa,$$

$$F_2(\mu_1) = \frac{(\Gamma^2 + 2\mu_1^2 + 2\Gamma\mu_1)}{\Gamma^2 + 3\mu_1^2}\kappa - 1.$$

It is easy to verify that $F_1(\mu_1)$ monotonically decreases when the control parameter μ_1 increases, thereby

$$F_1(\mu_1) \leq F_1(0) = 1 - \kappa = C(\rho_\infty^u).$$

Also, we can obtain that

$$F_2(\mu_1) \geq 1 - \kappa = C(\rho_\infty^u) \geq 0,$$

when μ_1 satisfies equation (20). In conclusion, we arrive at

$$C(\rho_\infty) = F_2(\mu_1) \geq 1 - \kappa = C(\rho_\infty^u).$$

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